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Dispersion-theoretical analysis of the nucleon electromagnetic form factors: Inclusion of time–like data[#1]

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Abstract

We update a recent dispersion–theoretical fit to the nucleon electromagnetic form factors by including the existing data in the time–like region. We show that while the time–like data for the proton can be described consistently with the existing world space–like data, this is not the case for the neutron. Another measurement of the process $e^+e^- \to \bar{n}n$ is called for. We furthermore sharpen the previous estimate of the separation between the perturbative and the non–perturbative regime, which is characterized by a scale parameter $\Lambda^2 \simeq 10\,\mathrm{GeV}^2$.

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1 Introduction

A detailed understanding of the electromagnetic form factors of the nucleon is not only of importance for revealing aspects of perturbative and non–perturbative nucleon structure but also serves as an important ingredient for precise tests of the Standard Model like e.g. in the Lamb shift measurements performed recently [1]. These form factors have been measured over a wide range of space–like momentum transfer squared, $t = 0 \ldots -35\,\mathrm{GeV}^2$ but also in the time–like region either in $\bar{p}p$ annihilation or in $e^+e^- \to \bar{p}p, \bar{n}n$ collisions [2]. In particular, the FENICE experiment [3] has for the first time measured the (magnetic) neutron form factor. These data and the corresponding ones for the proton seem to indicate a new resonance at $\sqrt{t} = 1.85\,\mathrm{GeV}$, which, however, is not fully consistent with the precise data from LEAR. For a comprehensive summary, see Ref.[4]. The data (space- and time–like) can be analyzed in a largely model–independent fashion by means of dispersion relations. In Ref.[5] a new dispersion theoretical analysis of the nucleon form factors has been performed based on the current world data basis for space–like momenta, i.e. for four–momentum transfer squared $-t = Q^2 \geq 0$. This work improved upon the one of Höhler et al. [6] in various respects. These are the implementation of the constraints from perturbative QCD (pQCD) at large momentum transfer, the inclusion of the recent neutron–atom scattering length determination to constrain the neutron charge radius and, of course, the inclusion of new data at low, moderate and high momentum transfer (as listed in [5]). However, the data for time–like momentum transfer ($t > 0$) have not been accounted for. In this paper, we will include these data and discuss the pertinent consequences. We stress that we do this without increasing the number of vector meson poles compared to Ref.[5] because only in that way we can sharpen the analysis presented there.

2 Formalism

Here, we briefly review the dispersion–theoretical formalism developed in Ref.[5]. Assuming the validity of unsubtracted dispersion relations for the four form factors $F_{1,2}^{(I=0,1)}(t)$, one separates the spectral functions of the pertinent form factors into a hadronic (meson pole) and a quark (pQCD) component as follows,

$$\begin{aligned} F_i^{(I=0)}(t) &= \tilde{F}_i^{(I=0)}(t)L(t) = \left[\sum_{I=0} \frac{a_i^{(I=0)}\, L^{-1}(M^2_{(I=0)})}{M^2_{(I=0)} - t}\right] \left[\ln\left(\frac{\Lambda^2 - t}{Q_0^2}\right)\right]^{-\gamma} \\ F_i^{(I=1)}(t) &= \tilde{F}_i^{(I=1)}(t)L(t) = \left[\tilde{F}_i^{\rho}(t) + \sum_{I=1} \frac{a_i^{(I=1)}\, L^{-1}(M^2_{(I=1)})}{M^2_{(I=1)} - t}\right] \left[\ln\left(\frac{\Lambda^2 - t}{Q_0^2}\right)\right]^{-\gamma} \end{aligned} \tag{1}$$

with

$$L(t) = \left[\ln\left(\frac{\Lambda^2 - t}{Q_0^2}\right)\right]^{-\gamma} \quad , \tag{2}$$

and $F_i^\rho(t) = \tilde{F}_i^\rho(t)\, L(t)^{-1}$ parametrizes the two–pion contribution (including the one from the ρ) in terms of the pion form factor and the P–wave $\pi\pi\bar{N}N$partial wave amplitudes in a parameter–free manner (for details see [5]). Furthermore, Λ separates the hadronic from the quark contributions, Q_0 is related to $\Lambda_{\rm QCD}$ and γ is the anomalous dimension,

$$F_i(t) \to (-t)^{-(i+1)} \left[\ln\left(\frac{-t}{Q_0^2}\right)\right]^{-\gamma} , \quad \gamma = 2 + \frac{4}{3\beta} \ , \quad i = 1, 2 \ , \tag{3}$$

for $t \to -\infty$ and β is the QCD β–function. In fact, the fits performed in [5] are rather insensitive to the explicit form of the asymptotic form of the spectral functions. To be specific, the additional factor $L(t)$ in Eq.(2) contributes to the spectral functions for $t > \Lambda^2$, i.e. in some sense parametrizes the intermediate states in the QCD regime, above the region of the vector mesons. The particular logarithmic form has been chosen for convenience. Obviously, the asymptotic behaviour of Eq.(2) is obtained by choosing the residues of the vector meson pole terms such that the leading terms in the $1/t$–expansion cancel. In practice, the additional logarithmic factor is of minor importance for the fit to the existing data. The number of isoscalar and isovector poles in Eq.(2) is determined by the stability crtiterium discussed in detail in [5]. In short, we take the minimum number of poles necessary to fit the data. Specifically, we have three isoscalar and three isovector poles. The best fit to the available space–like proton *and* neutron data will be called fit 1 in what follows. Inclusion of the time–like data leads to fit 2. We stress that we are keeping the number of meson poles fixed so as not to wash out the predictive power. Due to the various constraints (unitarity, normalizations, superconvergence relations) we end up having only three free parameters since two (three) of the isovector (isoscalar) masses can be identified with the ones of physical particles, see below and [5]. We also have performed fits with more poles, these will be mentioned but not discussed in detail below.

We should also make some remarks on the extraction of the time–like form factors to be discussed. At the nucleon–anti-nucleon threshold, one has

$$G_M(4m^2) = G_E(4m^2) \quad , \tag{4}$$

with $G_{E,M}$ the standard Sachs form factors

$$G_E = F_1 - \tau F_2 \, , \quad G_M = F_1 + F_2 \, , \quad \tau = \frac{-t}{4m^2} \ , \tag{5}$$

and at large momentum transfer one expects the magnetic form factor to dominate. From the data, one can not separate $|G_M|$ from $|G_E|$ so one has to make an assumption, either setting $|G_M| = |G_E| = |G|$ or $|G_E| = 0$. Most recent data are presented for the magnetic form factors [4] and we will proceed accordingly, i.e. we fit the magnetic form factors in the time–like region.

3 Results and discussion

Consider first fit 1. The isoscalar masses are $M_\omega = 0.782$ GeV, $M_\phi = 1.019$ GeV and $M_{S'} = 1.60$ GeV and the isovector ones are $M_{\rho'} = 1.68$ GeV, $M_{\rho''} = 1.45$ GeV and $M_{\rho'''} = 1.69$ GeV. We note that apart from the ρ' these can be identified with masses of observed vector meson resonances. The corresponding residua are given in table 1, with the lower index $i = 1, 2$ referring to the vector and tensor coupling, respectively, and the superscript $(n = 1, 2, 3)$ enumerates the nth pole. The isospin index $I = 0, 1$ is listed separately. The isovector residua differ somewhat from the ones given in [5] because we have updated the data basis [7][8][9]. The corresponding numbers on the nucleon radii and vector-meson–nucleon coupling constants given there are, however, not affected. The QCD parameters are $\gamma = 2.148$, $\Lambda^2 = 9.73$ GeV^2 and $Q_0^2 = 0.35$ GeV^2. The consequences for the extraction of the strange form factors are the same as in Ref.[10].

	$a_1^{(1)}$	$a_2^{(1)}$	$a_1^{(2)}$	$a_2^{(2)}$	$a_1^{(3)}$	$a_2^{(3)}$
Fit 1, $I = 0$	0.747	–0.122	–0.737	0.162	–0.039	–0.041
Fit 1, $I = 1$	189.3	35.90	–3.158	–6.476	–186.7	–30.92
Fit 2, $I = 0$	0.752	–0.121	–0.752	0.159	–0.020	–0.038
Fit 2, $I = 1$	–9.913	–4.731	13.01	0.263	–3.497	2.947

Table 1: Residues of the isoscalar $(I = 0)$ and the isovector $(I = 1)$ meson poles of the two fits discussed in the text.

The resulting Sachs form factors normalized to the canonical dipole fit are displayed by the dashed lines in Fig.1a.[#6] Fit 1 has a χ^2/datum of 1.06. It is worth mentioning that there is some inconsistency between the new precise data from NIKHEF [8] and Mainz [9] on one side and the ones from Bonn [12] on the other side. The fit prefers to stay slightly below the dipole approximation for $Q^2 \leq 1\,\mathrm{GeV}^2$.

We now turn to fit 2. As stated before, we include the existing time–like data but do not increase the number of meson poles. The number of data points is increased by 30. In the fit, we keep the physical masses fixed, i.e. we let the mass of the ρ' free and allow for variations in the pertinent residua. We find $M_{\rho'} = 1.40$ GeV, close to the mass of the ρ''. We also optimize the fit by letting Λ vary and find $\Lambda^2 = 12.0\,\mathrm{GeV}^2$. As noted in [5], the third isovector mass always tends to come out close to one of the other masses thus inducing some dipole–like structure. The isovector residua are mostly affected as shown in table 1. The corresponding normalized Sachs form factors for space–like t are shown as solid lines in Fig.1a and the (magnetic) time–like proton and neutron form factors are

[#6]In case of the neutron electric form factor we divide by $G_E^n(t)$ as given in Ref.[11] for the Paris potential with the normalization readjusted to give the exact value for the neutron charge radius.

displayed in Fig.1b (again by the solid lines). The χ^2/datum of the fit increases to 1.47. This is mostly due to the FENICE data for $|G_M^{p,n}(t)|$ for t close to the unphysical region at $t = 4m^2$. In particular, we are not able to reproduce the sharp increase of these data close to the unphysicsal region. We stress that the resulting time–like $|G_M^p(t)|$ agrees nicely with (most of) the LEAR data.

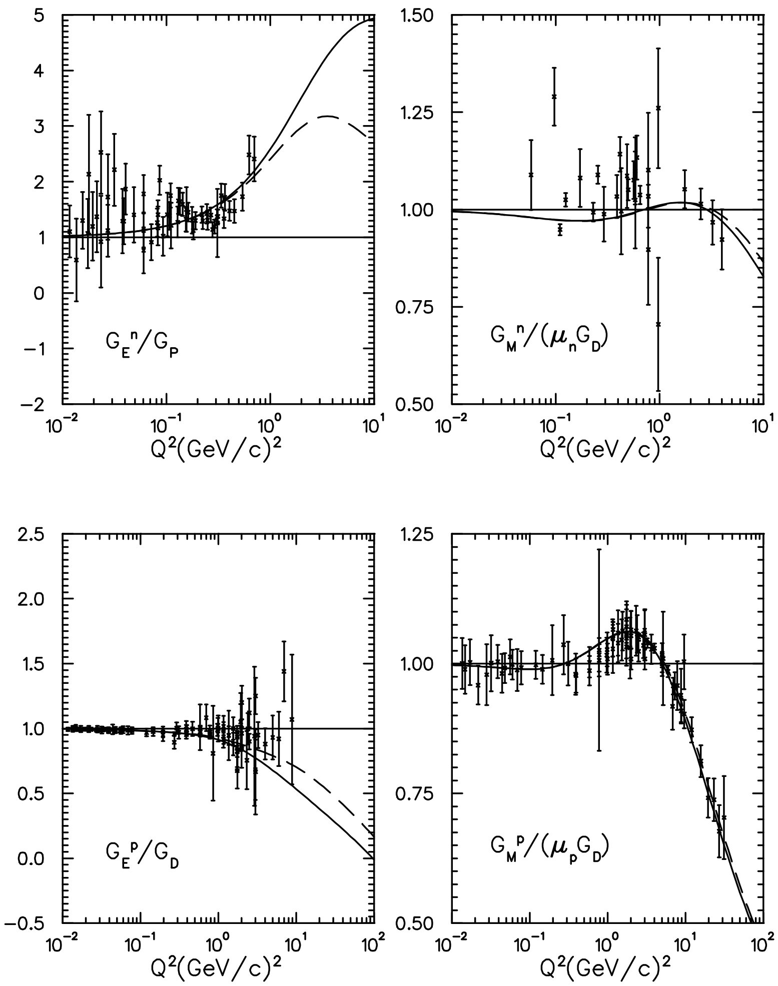

Fig. 1a: The space–like nucleon em form factors. Dashed lines: Fit 1, space–like data only. Solid lines: Fit 2, including also the time–like data.

Since with a three pole fit in the isoscalar and isovector channels we can not describe the FENICE data consistently, we have also performed fits with more poles. For example, if one adds a fourth isoscalar pole at the mass of the $\phi(1680)$, the fit does only improve marginally as shown by the dashed lines in Fig.1b. The χ^2/datum of this fit is 1.46. Similarly, a fourth isoscalar pole at 1.85 GeV does not lead to an improved fit. Considering the present situation of the data in the time–like region, in particular for the neutron, we believe that first a more solid data basis is needed before one has to increase the number of meson poles to get a better fit. This becomes more transparent if one removes the

FENICE neutron data from the data basis. Our standard three pole fit then leads to better χ^2/datum of 1.37. We also remark that a smaller neutron time–like form factor would be more consistent with pQCD estimates [4]. Some remarks on the sensitivity to the cut–off Λ^2 are in order. While in [5] one could vary its value between 5 and 15 GeV^2 without any drastic consequences for the fits, matters are differently here. To keep the χ^2/datum below 1.72, we can vary Λ^2 in the range from 9.3 to 16.3 GeV^2. We must stress, however, that there is still a sizeable contribution of the hadronic (pole) part of the spectral functions at $Q^2 > \Lambda^2$, compare e.g. Fig. 7 in [5]. This means that to test the predictions of pQCD one has to go to momentum transfer squared (much) larger than 20 GeV^2.

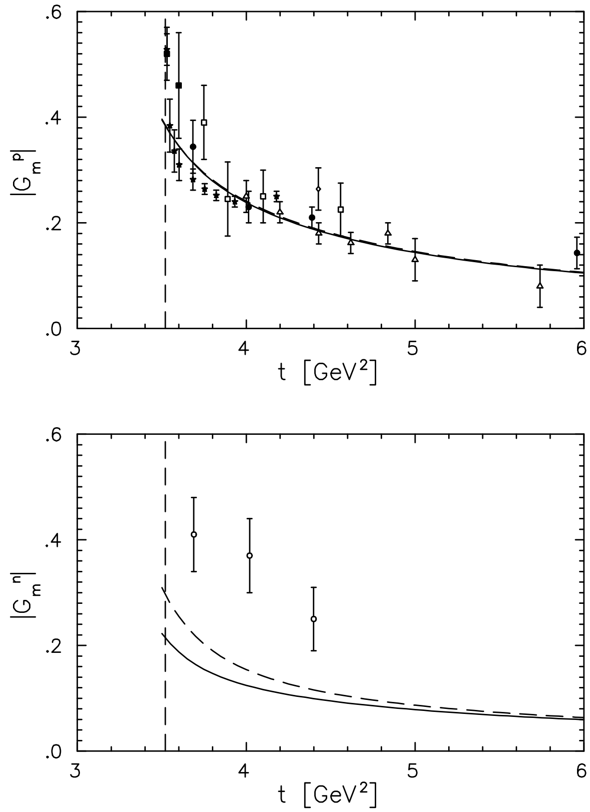

Fig. 1b: The time–like nucleon em form factors. Solid lines: Fit 2. Dashed lines: including in addition the $\phi(1680)$.

To summarize, we have extended the dispersion–theoretical fit of Ref.[5] by including the existing data for time–like four–momentum transfer. A fit with three isoscalar and three isovector poles does not allow to consistently describe all the time–like data while still giving a good description at space–like momentum transfer. Clearly, the discrepancies in the time–like data for $|G_M^p(t)|$ need to be resolved and additional experimental information on $|G_M^n(t)|$ is called for. Furthermore, a more refined treatment of the final–state interactions in $e^+e^- \to \bar{n}n$ might lead to a somewhat lower neutron form factor in the time–like region.

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